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Characterizing Microwave Planar Circuits Using the Coupled Finite-Boundary Element Method

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***Abstract*—A general approach is presented for the analysis of microwave planar circuits. The technique is particularly well suited to the analysis of circuits with complicated geometries and dielectric loads. The proposed technique is a hybrid, consisting of an amalgamation of the finite element and the boundary element techniques. The new technique can handle problems with mixed electric and magnetic walls, as well as complicated dielectric loads, such as those composed of ferrite materials. Computed and measured data for various complicated devices are compared, showing excellent agreement.**

I. Introduction

In the development of numerical techniques for analyzing the electromagnetic fields associated with microwave circuits, one has to strike a balance between accuracy and simplicity. Three dimensional full-wave analyses are often impractical because they result in long computational times and prohibitively large memories requirements. In contrast, a simple planar waveguide model can be used in the analysis. This model is particularly useful for solving microwave printed circuit problems. Several methods have been used in the past for the analysis of planar circuits. When the circuit pattern is as simple as a square, rectangle, circle, or annular section, field expansion in terms of resonant models can be used. A general procedure of this technique has been described by Okoshi [1]. When using this method, one must start by calculating the eigenfunctions and eigenvalues for the circuit being analyzed. This can be done analytically if the structure is a separable geometry; if the geometry is not separable, the numerical analysis can be carried out using the contour integral representation of the wave equation.

In practice, it is highly desirable for CAD software to have the capability of analyzing arbitrarily shaped planar circuits. The Finite Element Method (FEM) [2] has been used to handle the discontinuities of arbitrarily shaped planar circuits, even though the computational overhead is large. The boundary integral method or Boundary Element Method (BEM) [3] seems to offer the promise of greatly increased efficiency because it reduces the size of planar circuit problems from two-dimensions to one-dimension. However, when the planar circuits involve complicated or anisotropic dielectric loads, the difficulty of the solution increases quite considerably for one-dimensional algorithms.

In this paper, the coupled finite-boundary element method (CFBM), which was originally developed for waveguide discontinuities [4], is adopted for application to general microwave planar circuit structures. The circuits can include electric walls, magnetic walls, and complex dielectric loads. The CFBM has the merits of both the FEM and the BEM, and can be used to solve complicated problems without requiring excessive computer memory and computation time. Using this method, only the complex media subdomains, which may consist of lossy or anisotropic materials, need to be treated using FEM. Elsewhere, the BEM is used on the boundary to take into account the circuit configuration. Comparing the hybrid technique with the FEM (two-dimensional algorithm) and BEM (one-dimensional algorithm), one concludes that the CFBM can be considered to be a one and one-half dimensional algorithm for planar circuit analysis.

The validity of the application of CFBM to the analysis of planar circuits will be demonstrated with various illustrative examples, which include a cavity filter using metallic posts as inductive shunts, a microstrip disk filter with a dielectric load, and a planar ferrite circulator. The numerical results obtained with CFBM are compared with both measured results and published results. The comparisons are shown to be in good agreement.

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II. The Planar Circuit Model

There are various techniques that can be used for accurately determining both the equivalent waveguide width $W_{\text{eff}}(f)$ and equivalent filling dielectric constant $\epsilon_{\text{eff}}(f)$. Since they are well known they will not be discussed here for simplicity. When characterizing microstrip or strip line circuits using an equivalent cavity, surrounded by magnetic walls, it is necessary when establishing the equivalent dimensions to take into account the energy stored in the circuit's fringing field. The stored energy can be estimated by assuming that the electric and magnetic fields are constant throughout the increased volume ΔV and are equal to the field at the edge of the circuit. For instance, the increased volume ΔV for a circular disk is taken to be equal to the volume corresponding to the static fringe capacitance ΔC between the radius a and the equivalent radius a_{eq} of the fringe edge, where

$$a_{eq} = a\left\{1 + \frac{d}{\epsilon \pi a^2}\Delta C\right\}^{1/2}, \tag{1}$$

and the static fringe capacitance ΔC can be derived using Kirchhoff's equation. The equivalent parameters for other geometries can be obtained in a similar way.

It is usually assumed that the equivalent dimensions reflect only the characteristics of the fringing fields when unperturbed by inhomogeneities in the planar circuits, such as a dielectric loads or conductor posts. It can be shown, by comparing the numerical results with the experimental results, that this assumption is fairly reasonable because the equivalent dimensions are determined mostly by the fringing of the outermost fields.

III. General CFBM Formulation for Planar Circuits

The general planar circuit problem shown in Fig. 1 will now be addressed. An M-port device is assumed, where the boundary Γ_Q encloses the inhomogeneous subdomain; boundary Γ_0' is the possible electric wall in the circuit; and the boundary $\Gamma' = \Gamma_0' \cup \Gamma_Q \cup_{m=0}^{M} \Gamma_m$, completely encloses the remaining homogeneous domain.

When using the CFBM technique, complicated subdomains, consisting of dielectric or ferrite posts, can be treated using the

FEM, while the remaining homogeneous region, surrounded by $\Gamma_Q \cup \Gamma_0' \cup \Gamma'$ can be adequately looked after by using BEM. The detailed formulae and the procedure have been discussed in [4]. Therefore, in the interest of brevity, only the intermediate results are listed here.

By using the Galerkin procedure on the two-dimensional Helmholtz equation and discretizing the complicated subdomain using second-order finite elements, one can obtain the following matrix equation:

$$\begin{bmatrix} [A]_{Q,\Gamma_Q} & [A]_{Q,Q} \\ [A]_{\Gamma_Q,\Gamma_Q} & [A]_{\Gamma_Q,Q} \end{bmatrix} \begin{Bmatrix} \{E_z\}_{\Gamma_Q} \\ \{E_z\}_Q \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ [D] \left.\dfrac{\partial E_z}{\partial n}\right._{\Gamma_Q} \end{Bmatrix} \tag{2}$$

where $\{E_z\}_Q$ and $\{E_z\}_{\Gamma_Q}$ are, respectively, the vectors of the electric field E_z at the nodal points inside domain Q and on the boundary of Q; $[A]_{Q,Q}$, $[A]_{Q,\Gamma_Q}$, $[A]_{\Gamma_Q,Q}$, and $[A]_{\Gamma_Q,\Gamma_Q}$ are the submatrices associated with $[E_z]_Q$ and $\{E_z\}_{\Gamma_Q}$ in different combinations; and $[D]$ is a square coefficient matrix obtained through the integration of the shape function vector over the boundary Γ_Q. For the second order finite element, $[D]$ can be expressed as

$$[D] = \sum_e{}' \frac{l_e}{15} \begin{bmatrix} 2 & 1 & -0.5 \\ 1 & 8 & 1 \\ -0.5 & 1 & 2 \end{bmatrix} \tag{3}$$

where the l_e is the length of eth boundary element and Σ_e' extends over all the boundary elements. On the other hand, by discretizing the boundary integral equation, which describes the information for the remaining homogeneous domain, using so-called boundary elements, one can subsequently obtain the BEM matrix equation:

$$[H_0, H_0', H_1, H_2, \cdots H_M, H_{\Gamma_Q}] \begin{Bmatrix} \{E_z\} \\ \{E_z\}_{\Gamma_0'} \\ \{E_z\}_{\Gamma_1} \\ \vdots \\ \{E_z\}_{\Gamma_M} \\ \{E_z\}_{\Gamma_Q} \end{Bmatrix} = [G_0, G_0', G_1, G_2, \cdots G_M, G_{\Gamma_Q}] \begin{Bmatrix} \{\partial E_z/\partial n\}_{\Gamma_0} \\ \{\partial E_z/\partial n\}_{\Gamma_0'} \\ \{\partial E_z/\partial n\}_{\Gamma_1} \\ \vdots \\ \{\partial E_z/\partial n\}_{\Gamma_M} \\ \{\partial E_z/\partial n\}_{\Gamma_Q} \end{Bmatrix}. \tag{4}$$

The boundary elements used in (4) are similar to finite elements except that their dimensions are usually one less than that of the problem. In the present analysis, second order boundary elements are used for the sake of compatibility with the second order finite elements.

Assuming that the dominant TEM mode is incident from port j, a discretized relation of the electric field and its normal derivative at the planar waveguide ports has been found [3]:

$$\{E_z\}_{\Gamma_i} = 2\delta_{ij}\{f_{i0}\} + [Z]_i \left\{\frac{\partial E_z}{\partial n}\right\}_{\Gamma_i} \tag{5}$$

where

$$[Z]_i = -\sum_{m=0}^{\infty} \frac{1}{j\beta_{im}} \{f_{im}\} \sum_{e^i} \int_{e^i} f_{im}(y_0^{(i)}) \cdot \{N(x^{(i)} = 0, y_0^{(i)})\}\, dy_0^{(i)} \quad \cdots, M \tag{6}$$

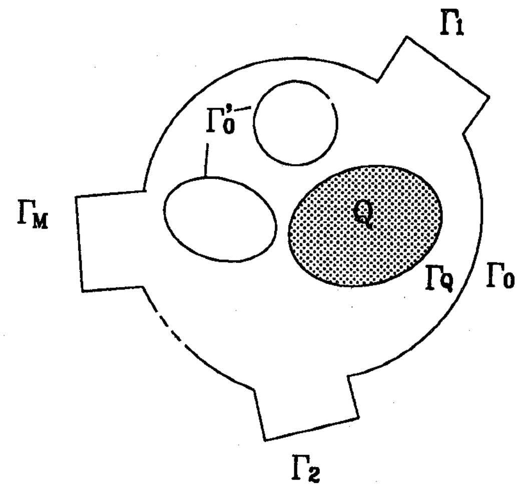

Fig. 1. The geometry of the problem and the computation regions.

and the components of the $\{f_{im}\}$ vector are the values of the mode function $f_{im}(y^{(i)})$ at the nodal points on Γ_i; Σ_{e^i} extends over the elements related to Γ_i; and $\{N\}$ is the second order boundary element shape function.

It should be realized that the boundary conditions that apply to our generalized problem consist of field continuity (compatibility), field normal derivative continuity (equilibrium), as well as the condition that $\partial E_z/\partial n$ equals to zero on the magnetic wall, and E_z vanishes on the conductor surfaces. The complete problem being addressed here can now be described finally in a matrix form, i.e., with the help of (2), (4), (5), and the boundary conditions; it is written in the form:

$$\begin{bmatrix} [H_0, H_1, H_2, \cdots H_M, H_Q, 0] & -[G_0; G_1, \cdots G_M, G_Q] \\ \begin{bmatrix} & 0 & \end{bmatrix} \begin{bmatrix} A \end{bmatrix} & \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & D \end{bmatrix} \\ \begin{bmatrix} 0 \end{bmatrix} \begin{bmatrix} 1 & & 0 \\ & 1 & \\ & & \ddots \\ 0 & & 1 \end{bmatrix} \begin{bmatrix} & 0. & \end{bmatrix} & \begin{bmatrix} -Z_1 & 0 \\ & \ddots \\ 0 & -Z_M \end{bmatrix} \begin{bmatrix} 0 \end{bmatrix} \end{bmatrix} \cdot \begin{Bmatrix} \{E_z\}_{\Gamma_0} \\ \{E_z\}_{\Gamma_1} \\ \vdots \\ \{E_z\}_{\Gamma_M} \\ \{E_z\}_{\Gamma_Q} \\ \{E_z\}_Q \\ \dfrac{\partial E_z}{\partial n}\Big|_{\Gamma_{0'}} \\ \dfrac{\partial E_z}{\partial n}\Big|_{\Gamma_1} \\ \vdots \\ \dfrac{\partial E_z}{\partial n}\Big|_{\Gamma_M} \\ \dfrac{\partial E_z}{\partial n}\Big|_{\Gamma_Q} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 2\delta_{ij} f_{i0} \\ 0 \\ \vdots \\ 0 \end{Bmatrix}. \tag{7}$$

In the above equation, [1] is an identity matrix; [0] is an empty matrix, $\{E_z\}_{\Gamma_i}$ and $\{\partial E_z/\partial n\}_{\Gamma_i}$ $(i = 0, 0', 1, 2, \cdots M, Q)$ correspond to electric fields and their normal derivatives at the nodal points related to boundary $\Gamma_0, \Gamma_{0'}, \Gamma_1, \Gamma_2, \cdots, \Gamma_M, \Gamma_Q$, respectively; and $\{0\}$ is a null vector.

The solution of the matrix equation determines the scattered electric field distribution across each planar waveguide port. Using the orthogonality of the modes in a planar waveguide, the scattering parameters of the fundamental mode can be easily determined.

IV. Numerical Implementation and Examples

The algorithm described in the preceding section has been implemented as an user-friendly computer package, named PLANAR MICROWAVE CIRCUIT SIMULATOR. This software package has the advantage of requiring only short CPU computer time, as well as having a small memory requirement. This then allows the package to be run on IBM PCs or compatibles. The full-screen graphics interface makes it easy to input the device geometry [5].

Example 1: A Microstrip Line with Conductor Posts (Cavity Filter)

When the substrate in a circuit is homogeneous, the general formula, i.e. (7), degenerates into the BEM-based algorithm. A typical example of this special case is a four-post band-pass filter, using half-wavelength sections as series resonators and shunt posts, as shown in Fig. 2. The filter was originally synthesized using Chebyshev filter theory with a ripple level of 1.0 dB, and constructed using a four-post structure in a Duroid substrate with thickness = 0.49 mm and $\epsilon_r = 2.43$. However, in the present analysis, the filter is considered as a completely integrated system consisting of four circular electric walls and magnetic side walls. The CFBM result agrees well with the results of the multipole expansion method (not shown here) and the measured results are presented in [6].

Example 2: Microstrip Disk Filter with Dielectric Load

A microstrip circular disk filter is considered as an example of a planar circuit with an arbitrary shape. In this study, the effective radius $R_{\text{eff}} = 22.8$ mm is derived from the value of the original radius, i.e. $R = 21.58$ mm. In this example, a square dielectric load with $\epsilon_r = 10.2$ is placed at the center of a disk. The comparison of the predicated and measured results in Fig. 3 indicates that the CFBM is applicable to complex microstrip planar devices. Strictly speaking, fringing effects depend on the resonant mode and can be taken into account more accurately by considering an equivalent model for each mode. This mode dependency is evident in the discrepancy between measurements and simulations at higher frequency. Considering the inevitable radiation loss that will be experienced by this open structure, the small discrepancy can be expected, even though the TRL calibration technique was used for the measurements.

Example 3: A 3-Port Microwave Planar Ferrite Circulator

As an example of a planar circuit of thin substrate with an anisotropic dielectric load, a Y junction with a TT1-109 triangular ferrite post shown in Fig. 4 is considered. The widths of the three planar waveguide ports are the same (i.e. $W_1 = W_2 = W_3 = 8.2$ mm) and the substrate constant is, $\epsilon_r = 2.2$. In the analysis, both magnetic losses and dielectric losses are neglected. The S-parameters obtained using the CFBM are shown in Fig. 4, where it is observed the device acts like a wideband isolator (circulator).

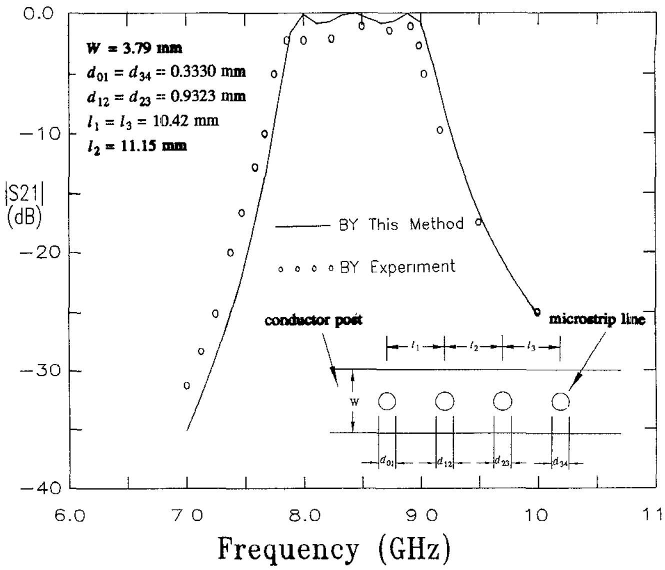

Fig. 2. Computed and measured $|S_{21}|$ for a four-post band-pass direct-coupled cavity filter.

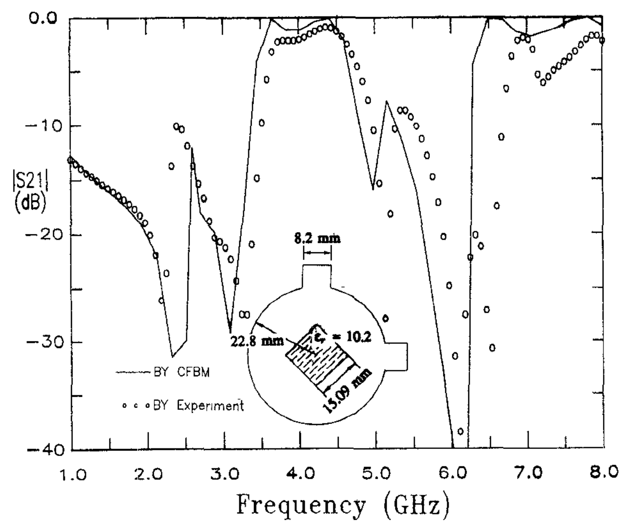

Fig. 3. Computed and measured transmission coefficients for a microstrip circular disk filter with a square dielectric load.

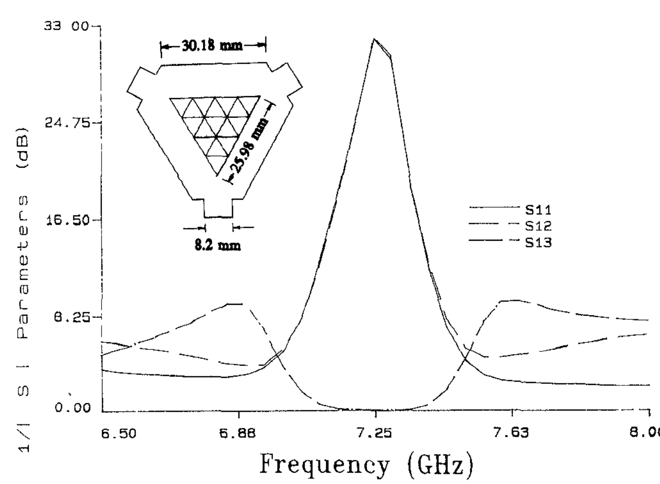

Fig. 4. Scattering parameters of a planar Y-junction circulator with an equilateral triangular ferrite post.

In this example, only 16 triangular second order finite elements are used to obtain a convergent result.

V. Conclusions

A generalized coupled finite and boundary element method has been shown to be applicable to microwave planar circuit problems. The planar waveguide model is used in developing the technique. The technique takes advantage of the strengths of the finite element and boundary element methods. Thus, it can handle complicated and arbitrarily shaped planar circuits with a small computational overhead. The validity of the method was confirmed by comparing the CFBM results, either with published results or with experimental results. The performance of a Y-junction circulator with an equilateral triangular ferrite post was also investigated. For all the numerical examples, the power conservation condition has been found to be satisfied to an accuracy of $\pm 10^{-5}$ to $\pm 10^{-4}$ within the frequency band of the dominant mode.

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A Modified Transverse Resonance Method for the Analysis of Multilayered, Multiconductor Quasiplanar Structures with Finite Conductor Thickness and Mounting Grooves

Jun-Wu Tao

***Abstract*—A modified transverse resonance method is presented for analyzing generalized multilayered, multiconductor quasiplanar structures with practical parameters such as finite conductor thickness and mounting grooves. Recurrence relations are obtained by using network theory, for obtaining the overall transverse equivalent network, while the discontinuity involving finite thickness metal sheet and mounting groove is carried out by field-theory based multimodal variational formulation. The frequency behavior of propagating, evanescent and complex modes are obtained for several commonly used quasiplanar lines, showing good agreement with published results. Furthermore, the leaky-wave study is carried out for open structures, since the open condition can be included in this formulation without difficulties.**

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I. Introduction

Various planar and quasiplanar structures play a determinant role in the realization of MIC's and MMIC's, which found growing interest in microwave and millimeter-wave subsystems design [1], [2]. Many rigorous, full-wave analysis techniques have been developed for their characterization, among which the most popular will be the spectral domain approach [3], [4], the integral equation technique [5]–[8], and various mode-matching techniques [9]–[12]. It has been shown that the spectral domain approach, as well as the integral equation technique, are numerically efficient. On the other hand, the mode-matching techniques are more versatile since the practical parameters of real quasiplanar structures, such as the metallization thickness and the mounting grooves, can be easily taken into account; nevertheless, more numerical effort will be paid because of relatively large matrix involved.

We present here another modified transverse resonance technique. Contrary to the existing one in which the problem is formulated entirely by the field theory [9], [11], we use both the field and network theories, as in the classical transverse resonance method [13]. By considering the generalized quasiplanar structure as cascaded parallel-plate waveguides, each homogeneous region will be characterized by a transmission matrix, instead of the TEM line in [13], and the junction between two parallel-plate waveguides characterized by a multiport, instead of a shunt admittance. The characteristic equation can then be easily found by applying the resonance condition. The solution is obtained only by applying the well known and easy to use network theory, except for the reduced impedance matrix of parallel-plate waveguide junctions which will be achieved by a rigorous multimodal variational method [14]. Furthermore, as we can choose the size of each impedance matrix, that is, the number of coupling modes in each parallel-plate waveguide, without affecting the accuracy of remaining matrix elements, as explained in [14], the resultant matrix size can be much smaller than the number of eigenmodes considered, providing a convenient way to accelerate the numerical computations. The classical transverse resonance method is then a particular case of this formulation when only the dominant TEM mode is considered. A quasiplanar structure simulation program has been developed on a personal computer, and applied to several commonly used planar and quasiplanar waveguides by considering both the metallization thickness and the mounting groove. Original results such as the complex and backward modes in the suspended microstrip are presented, as well as the leaky modes in the open quasiplanar structure which is very useful in the novel-type millimeter-wave antenna design [15], [16]. The latter has been compared to the published results concerning a leaky-wave microstrip antenna [15], showing good agreement.

II. Modified Transverse Resonance Method

Fig. 1(a) shows a multilayered, multiconductor quasiplanar guiding structure, with its equivalent transverse network model in Fig. 1(b). The reduced impedance matrix of each multiport, which